Reconciliation of effective potential calculations of the lower bound for the Higgs boson mass

R.S.Willey*

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There are three recent calculations of the lower bound for the mass of the Higgs boson of the Standard Model, based on the position of the minimum of the effective potential. [1] (I will refer to these as S,AI,CEQ) In all cases, the result depends on m_t, α_s, and on the cutoff scale beyond which new physics is assumed to set in. For a high scale ($\sim m_{PL}$) cutoff, the results are in the range 130 to 140 GeV, for $m_t = 174, \alpha_s = .118$. Five to ten GeV is also the claimed uncertainty in the high scale case. However, for the low scale cutoff ($\sim$ one TeV), there is a substantial discrepancy. AI obtain for the lower bound 72 GeV; CEQ obtain 55 GeV. (S does not give a low scale result for these values of m_t, α_s). [2] The claimed uncertainty in the low scale calculations is only a few GeV. If this discrepancy persists, the reliability of the general procedure is placed in doubt.

I believe that I have traced the origin of the discrepancy to the treatment of the relation between the perturbative pole mass and the the$\bar{M}S$ mass of the top quark in CEQ. Equation (18) of CEQ is the relation between these quantities in QCD; but when the Higgs-Yukawa sector is included there are additional contributions which are larger

*Physics Dept, University of Pittsburgh, Pittsburgh PA 15260; E-Mail:willey@vms.cis.pitt.edu

than the QCD contribution and of the opposite sign.[3]

$$m^* = m\{1 + \frac{\alpha_s}{\pi} C_F + \frac{y^2}{16pi^2}\frac{1}{2}\Delta(r) + \delta\overline{\zeta}_v\}$$

$$\delta\overline{\zeta}_v = \frac{1}{16\pi^2}[3\lambda(1 - \ln\frac{M^2}{m^2}) - 2N_c y^2 \frac{m^2}{M^2} + \frac{3}{2}g_2^2\frac{M_W^2}{m^2}(1 - \ln\frac{M_W^2}{m^2}) + \frac{3}{4}\frac{g_2^2}{c^2}\frac{M_Z^2}{m^2}(1 - \ln\frac{M_Z^2}{m^2})] \quad (1)$$

$$\Delta(r) = -\frac{1}{2} + \int_0^1 dx(2-x)\ln((1-x)r^2 + x^2)$$

In these equations m^* is the top pole mass, m is the $\bar{M}S$ top mass,and M is the Higgs mass. r is the ratio $\frac{M}{m}$ and y is the top Yukawa coupling ($y = \sqrt{2}\frac{m}{v}$). $\delta\overline{\zeta}_v$ is the finite part of the one-loop tadpole contribution which is not removed by $\bar{M}S$ subtraction. [3](There are also small contributions proportional to the gauge coupling constants g_2^2, g_1^2 which are neglected in the first line) Note the term proportional to $\frac{m^2}{M^2}$. One sees immediately that there will be large corrections for the low scale (lighter Higgs) case and much smaller correction for the high scale case, which is what is required.

CEQ use the QCD relation to fix the values $m(m^*), y(m^*)$ for the $\bar{M}S$ top mass and Yukawa coupling constant, $m(t), y(t)$, for a given input top mass e.g. $m^* = 174$. The actual calculation of CEQ, given this input, is rather complicated, but we can estimate the effect of using (1) in place of CEQ eq (18) to make this connection, as follows: First, for some input m^*, solve (1) for $m(m^*)$.Then solve CEQ eq (18) for a new $m^{*'}$ required to produce that same m. Then the estimate for the corrected lower bound Higgs mass is the value read from fig 5 or fig 6 of CEQ for $m^{*'}$. There is one new feature. The more complicated (1) depends on the Higgs mass M. So one has to guess the output M and put it into (1) and do the calculation, and then iterate if neccessary. For the low scale cutoff calculation, we put $M = 77$ into (1) and find $m(174) = 198$. We substitute that value into eq (18) of CEQ to find what $m^{*'}$ CEQ would have to use to get that initial value. The answer is $m^{*'} = 207$. By extrapolation of fig 5 of CEQ to $m^{*'} = 207$, the corrected lower bound for the Higgs mass looks to be close to 77 GeV. For the high scale calculation, we start by guessing $M = 150$ to put in (1). Now the QCD and nonQCD terms almost exactly cancel, m(174) = 174. Then $m^{*'} = 182$. For this value of $m^{*'}$, fig 6 of CEQ gives the lower bound to be 150 GeV. This is to be compared to values 141 GeV from S and 135 Gev from AI.

These numbers are not to be taken entirely seriously. They are the results of a simple patch job, not a real calculation. (There is also the problem of reading data from photographically reduced figures). However I do take them as strong indication that the real results of the CEQ calculation may be consistent with those of S and AI, with uncertainties which are only a little larger than claimed (and are larger for the large extrapolation involved in the high scale case than in the low scale limitt).

References

[1] M.Sher,Phys.Lett.B 331(1994) 448; G.Altarelli,G.Isidori, Phys.Lett. B 357(1994) 141; J.A.Casas,J.R.Espinosa,M.Quiros,Phys.Lett. B 342(1995) 171.

[2] I have recently given an alternative derivation of a lower bound for the Higgs mass which does not make use of the effective potential. For the same input parameters the results are 72 GeV for the low scale cutoff and 148 GeV for the high scale cutoff. hep-ph9512226.

[3] A.I.Bochkarev and R.S.Willey,Phys.Rev. D51(1995) R2049; R.Hempfling and B.Kniehl,Phys.Rev. D51(1995) 1386. Eq (14) in BW has to have restored a term $-6\hat{\lambda}\ln\frac{M^2}{m^2}$, for $\mu = m$. This is described in detail in an unpublished manuscript of Bochkarev and Willey.